JEE (Main)-2026 Session-1

Question Paper with Solutions

(Mathematics, Physics, And Chemistry)

21 January 2026 Shift – 2

Time: 3 hrs.   **M.M: 300**

IMPORTANT INSTRUCTIONS:

(1) The test is of 3 hours duration.

(2) This test paper consists of 75 questions. Each subject (PCM) has 25 questions. The maximum marks are 300.

(3) This question paper contains Three Parts. Part-A is Physics, Part-B is Chemistry and Part-C is Mathematics. Each part has only two sections: Section-A and Section-B.

(4) Section - A: Attempt all questions.

(5) Section - B: Attempt all questions.

(6) Section - A (01 - 20) contains 20 multiple choice questions which have only one correct answer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.

(7) Section - B (21 - 25) contains 5 Numerical value-based questions. The answer to each question should be rounded off to the nearest integer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.

MATHEMATICS

SECTION-A

1. The positive integer n, for which the solutions of the equation

$$x(x+2) + (x + 2)(x + 4) + \ldots +(x + 2n - 2)(x+2n) = \frac{8n}{3}$$

are two consecutive even integers, is :-

(1) 3   (2) 6

(3) 12   (4) 9

Ans. (1)

Sol. $x(x+2) + (x+2)(x+4) + \ldots+ (x+2n-2)(x+2n) = \dfrac{8n}{3}$

$$\Rightarrow \sum_{r=1}^{n}(x + 2r - 2)(x + 2r) = \frac{8n}{3}$$

$$nx^2 + 2x\sum_{r=1}^{n}(2r-1) + 4\sum_{r=1}^{n}r(r-1) = \frac{8n}{3}$$

$$nx^2 + 2x \cdot n^2 + \frac{4n(n^2-1)}{3} - \frac{8n}{3} = 0$$

$$x^2 + 2nx + \frac{4(n^2-1)}{3} - \frac{8}{3} = 0 \begin{smallmatrix}\alpha\\\beta\end{smallmatrix}$$

$$\because |\alpha - \beta| = 2 \Rightarrow \frac{\sqrt{D}}{|a|} = 2 \Rightarrow D = 4$$

$$\Rightarrow 4n^2 - 4\left(4\frac{(n^2-1)}{3} - \frac{8}{3}\right) = 4$$

$$\Rightarrow n^2 - \frac{4n^2}{3} = -3$$

$$\Rightarrow n^2 = 9$$

$$\Rightarrow n = 3$$

2. Let $f : \mathbf{R} \to \mathbf{R}$ be a twice differentiable function such that $f''(x) > 0$ for all $x \in \mathbf{R}$ and $f'(a-1) = 0$, where a is real number. Let $g(x) = f(\tan^2 x - 2\tan x + a)$, $0 < x < \dfrac{\pi}{2}$.

Consider the following two statements :

(I) g is increasing in $\left(0, \dfrac{\pi}{4}\right)$

(II) g is decreasing in $\left(\dfrac{\pi}{4}, \dfrac{\pi}{2}\right)$

Then,

(1) Neither (I) nor (II) is True

(2) Only (II) is True

(3) Only (I) is True

(4) Both (I) and (II) are True

Ans. (1)

Sol. $g(x) = f((\tan x - 1)^2 + a - 1)$

$g'(x) = f'((\tan x - 1)^2 + a - 1) \cdot 2(\tan x - 1)\sec^2 x$

$\because \ f'(a-1) = 0$ and $f''(x) > 0$

$\therefore f'((\tan x - 1)^2 + a - 1) > 0$

$g'(x) > 0$ if $(\tan x - 1) > 0$

g is increasing in $x \in \left(\dfrac{\pi}{4}, \dfrac{\pi}{2}\right)$

$g'(x) < 0$ if $\tan x - 1 < 0$

g is decreasing in $x \in \left(0, \dfrac{\pi}{4}\right)$

3. Let $f(x) = x^3 + x^2 \ f'(1) + 2x \ f''(2) + f'''(3)$, $x \in$ R. Then the value of $f'(5)$ is :

(1) $\dfrac{62}{5}$ (2) $\dfrac{657}{5}$

(3) $\dfrac{2}{5}$ (4) $\dfrac{117}{5}$

Ans. (4)

Sol. $f'(x) = 3x^2 + 2xf'(1) + 2f''(2)$

$f''(x) = 6x + 2f'(1)$

$f''(2) = 12 + 2f'(1)$

$\therefore f'(x) = 3x^2 + 2xf'(1) + 2(12 + 2f'(1))$

$f'(x) = 3x^2 + 2(x + 2)f'(1) + 24$

Putting, $x = 1$

$f'(1) = 3 + 6f'(1) + 24$

$-5f'(1) = 27 \Rightarrow f'(1) = \dfrac{-27}{5}$

$\therefore f''(2) = 12 + 2\left(\dfrac{-27}{5}\right) = 12 - \dfrac{54}{5} = \dfrac{6}{5}$

$\therefore f'(x) = 3x^2 - \dfrac{54}{5}x + \dfrac{12}{5}$

$\therefore f'(5) = 75 - 54 + \dfrac{12}{5} = \dfrac{117}{5}$

4. In the line $\alpha x + 4y = \sqrt{7}$, where $\alpha \in R$, touches the ellipse $3x^2 + 4y^2 = 1$ at the point P in the first quadrant, then one of the focal distances of P is :

(1) $\dfrac{1}{\sqrt{3}} - \dfrac{1}{2\sqrt{11}}$ (2) $\dfrac{1}{\sqrt{3}} + \dfrac{1}{2\sqrt{5}}$

(3) $\dfrac{1}{\sqrt{3}} - \dfrac{1}{2\sqrt{5}}$ (4) $\dfrac{1}{\sqrt{3}} + \dfrac{1}{2\sqrt{7}}$

Ans. (4)

Sol. $\alpha x + 4y - \sqrt{7} = 0$ touches $3x^2 + 4y^2 = 1$

$\therefore c^2 = a^2m^2 + b^2$

$\dfrac{7}{16} = \dfrac{1}{3} \times \dfrac{\alpha^2}{16} + \dfrac{1}{4} \Rightarrow \alpha = 3, -3$

Tangent is $3x + 4y - \sqrt{7} = 0$

Let the point of contact is $P(x_1 y_1)$

$\therefore$ Tangent is $3xx_1 + 4yy_1 = 1$

$\therefore \dfrac{3x_1}{3} = \dfrac{4y_1}{4} = \dfrac{1}{\sqrt{7}}$ $\therefore P\left(\dfrac{1}{\sqrt{7}}, \dfrac{1}{\sqrt{7}}\right)$

$e = \sqrt{1 - \dfrac{3}{4}} = \dfrac{1}{2}$
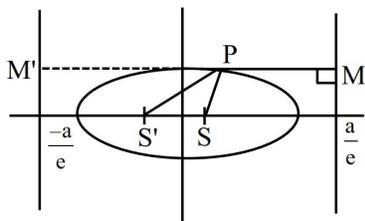
$PS = e\,(PM)$

$= e\left(\dfrac{a}{e} - \dfrac{1}{\sqrt{7}}\right)$

$= \dfrac{1}{2}\left(\dfrac{2}{\sqrt{3}} - \dfrac{1}{\sqrt{7}}\right) = \dfrac{1}{\sqrt{3}} - \dfrac{1}{2\sqrt{7}}$

$PS' = e\,(PM') = \dfrac{1}{2}\left(\dfrac{a}{e} + \dfrac{1}{\sqrt{7}}\right) = \dfrac{1}{2}\left(\dfrac{1}{\sqrt{7}} + \dfrac{2}{\sqrt{3}}\right)$

$= \dfrac{1}{\sqrt{3}} + \dfrac{1}{2\sqrt{7}}$

5. Let $y^2 = 12x$ be the parabola with its vertex at O. Let P be a point on the parabola and A be a point on the x-axis such that $\angle OPA = 90°$. Then the locus of the centroid of such triangles OPA is :

(1) $y^2 - 6x + 4 = 0$ (2) $y^2 - 9x + 6 = 0$

(3) $y^2 - 2x + 8 = 0$ (4) $y^2 - 4x + 8 = 0$

Ans. (3)

Sol.
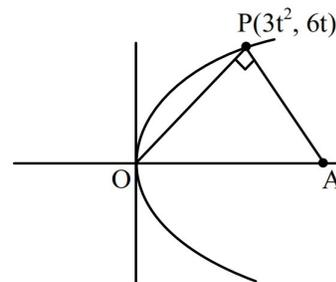
$m_{AP} = \dfrac{-t}{2}$

Equation of AP is

$y - 6t = \dfrac{-t}{2}(x - 3t^2)$

Put $y = 0 \Rightarrow x = 12 + 3t^2$

$\Rightarrow A(12 + 3t^2, 0)$

Let centroid of $\triangle$ OPA be G(h, k)

$\Rightarrow 3h = 0 + 3t^2 + 12 + 3t^2$

$3k = 0 + 6t + 0$

$\Rightarrow t = \dfrac{k}{2}, \quad h = 2t^2 + 4$

$\Rightarrow h = 2\dfrac{k^2}{4} + 4$
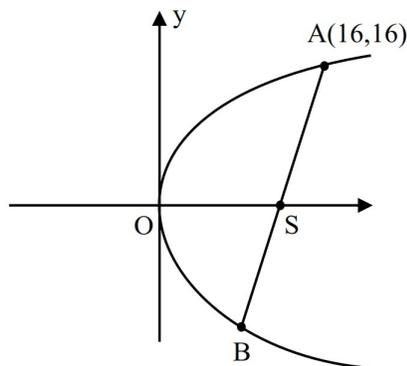
$\Rightarrow$ Locus of (h, k) is

$y^2 = 2x - 8$

6. Let one end of a focal chord of the parabola $y^2 = 16x$ be (16, 16). If $P(\alpha, \beta)$ divides this focal chord internally in the ratio 5 : 2, then the minimum value of $\alpha + \beta$ is equal to :

(1) 22   (2) 7
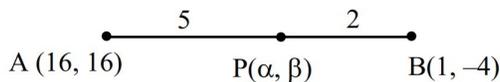
(3) 5   (4) 16

Ans. (2)

Sol. $y^2 = 16x$

∵ parameter of point A is t = 2

⇒ Parameter of point B is $t = -\dfrac{1}{2}$

⇒ Coordinates of B is (1, –4)

Case 1 :

A (16, 16)   P(α, β)   B(1, –4)

$\alpha = \dfrac{5+32}{7} = \dfrac{37}{7}$

$\beta = \dfrac{-20+32}{7} = \dfrac{12}{7}$

⇒ α + β = 7

Case 2 :

A(16, 16)   P(α, β)   B(1, –4)
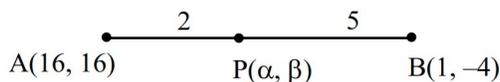
$\alpha = \dfrac{2+80}{7},$ $\beta = \dfrac{-8+80}{7}$

α + β = 22

So minimum value of α + β = 7

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7. Let the line L pass through the point (–3, 5, 2) and make equal angles with the positive coordinate axes. If the distance of L from the point (–2, r, 1) is $\sqrt{\dfrac{14}{3}}$, then the sum of all possible values of r is :

(1) 12   (2) 16

(3) 6   (4) 10

Ans. (4)

Sol. Equation line is : $\dfrac{x+3}{1} = \dfrac{y-5}{1} = \dfrac{z-2}{1} = \lambda$

∴ General point R on line is R(λ–3, λ+5, λ+2)

$\overrightarrow{PR} \equiv (\lambda - 1, \lambda + 5 - r, \lambda + 1)$

Now $\overrightarrow{PR} \cdot \vec{d} = 0$

⇒ (λ – 1)1 + (λ + 5 – r)1 + (λ + 1)1 = 0
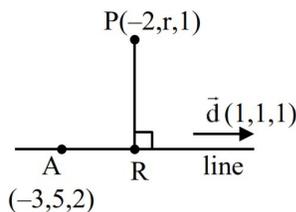
⇒ 3λ – r + 5 = 0

⇒ $\lambda = \dfrac{r-5}{3}$

∴ $R \equiv \left(\dfrac{r-5}{3} - 3, \dfrac{r-5}{3} + 5, \dfrac{r-5}{3} + 2 \right)$

$R \equiv \left(\dfrac{r-14}{3}, \dfrac{r+10}{3}, \dfrac{r+1}{3} \right)$

Now

$PR = \sqrt{\dfrac{14}{3}} \Rightarrow (PR)^2 = \dfrac{14}{3}$

$\Rightarrow \left(\dfrac{r-14}{3} + 2 \right)^2 + \left(\dfrac{r+10}{3} - r \right)^2 + \left(\dfrac{r+1}{3} - 1 \right)^2 = \dfrac{14}{3}$

$\Rightarrow \dfrac{(r-8)^2}{9} + \dfrac{(10-2r)^2}{9} + \dfrac{(r-2)^2}{9} = \dfrac{14}{3}$

⇒ $(r^2 - 16r + 64) + (100 + 4r^2 - 40r) + (r^2 - 4r + 4) = 42$

⇒ $6r^2 - 60r + 126 = 0$

⇒ $r^2 - 10r + 21 = 0$

⇒ r = 3, 7

sum of possible value of r is = 10

8. Let the line L_1 be parallel to the vector $-3\hat{i}+2\hat{j}+4\hat{k}$ and pass through the point $(2, 6, 7)$ and the line L_2 be parallel to the vector $2\hat{i}+\hat{j}+3\hat{k}$ and pass through the point $(4, 3, 5)$. If the line L_3 is parallel to the vector $-3\hat{i}+5\hat{j}+16\hat{k}$ and intersects the lines L_1 and L_2 at the points C and D, respectively, then $\left|\overrightarrow{CD}\right|^2$ is equal to :

(1) 171   (2) 290

(3) 312   (4) 89

Ans. **(2)**

Sol. $L_1 : \dfrac{x-2}{-3}=\dfrac{y-6}{2}=\dfrac{z-7}{4}$

Point C on L_1: $(-3\lambda_1 + 2, 2\lambda_1 + 6, 4\lambda_1 + 7)$

$L_2 : \dfrac{x-4}{2}=\dfrac{y-3}{1}=\dfrac{z-5}{3}$

Point D on L_2: $(2\lambda_2 + 4, \lambda_2 + 3, 3\lambda_2 + 5)$

Dr's of line L_3 :

$L_3 : \dfrac{2\lambda_2 + 3\lambda_1 + 2}{-3}=\dfrac{\lambda_2 - 2\lambda_1 - 3}{5}=\dfrac{3\lambda_2 - 4\lambda_1 - 2}{16}$

$\lambda_1 = -3,\ \lambda_2 = 2$

C (11, 0, –5)

D (8, 5, 11)

$\left|\overrightarrow{CD}\right|^2 = 3^2 + 5^2 + 16^2 = 290$

9. Let α and β be the roots of equation $x^2 + 2ax + (3a + 10) = 0$ such that $\alpha < 1 < \beta$. Then the set of all possible values of a is :

(1) $\left(-\infty, \dfrac{-11}{5}\right)\cup(5,\infty)$ (2) $(-\infty,-2)\cup(5,\infty)$

(3) $(-\infty,-3)$ (4) $\left(-\infty,\dfrac{-11}{5}\right)$

Ans. **(4)**

Sol. $\because\ \alpha < 1 < \beta$

$f(1) < 0$

$\Rightarrow 1 + 2a + (3a + 10) < 0$

$\Rightarrow 5a + 11 < 0$

$a < \dfrac{-11}{5}$

$\therefore\ a \in \left(-\infty, \dfrac{-11}{5}\right)$

10. A random variable X takes values 0, 1, 2, 3 with probabilities $\dfrac{2a+1}{30}, \dfrac{8a-1}{30}, \dfrac{4a+1}{30}$, b respectively, where a, b $\in \mathbf{R}$. Let μ and σ respectively be the mean and standard deviation of X such that $\sigma^2 + \mu^2 = 2$. Then $\dfrac{a}{b}$ is equal to :

(1) 30   (2) 3

(3) 60   (4) 12

Ans. **(3)**

Sol.

x	0	1	2	3
p(x)	$\dfrac{2a+1}{30}$	$\dfrac{8a-1}{30}$	$\dfrac{4a+1}{30}$	b

$\sigma^2 = \sum x_i^2 p(x_i) - \mu^2$

$\sigma^2 + \mu^2 = \sum x_i^2 p(x_i)$

$= 0 + 1\left(\dfrac{8a-1}{30}\right) + 4\left(\dfrac{4a+1}{30}\right) + 9b$

$\Rightarrow \dfrac{24a + 270b + 3}{30} = 2$

24 a + 270 b = 57

8a + 90b = 19 ….(1)

Also

$\sum p(i) = 1$

$\dfrac{2a+1}{30} + \dfrac{8a-1}{30} + \dfrac{4a+1}{30} + b = 1$

14 a + 30 b = 29 …(2)

Solving (1) & (2)

$a = 2,\quad b = \dfrac{1}{30},\ \dfrac{a}{b} = 60$

11. If the area of the region $\{(x, y) : 1 - 2x \leq y \leq 4 - x^2,$ $x \geq 0, y \geq 0\}$ is $\dfrac{\alpha}{\beta}$, $\alpha, \beta, \in N$, gcd $(\alpha, \beta) = 1$, then the value of $(\alpha + \beta)$ is :

(1) 73   (2) 85

(3) 91   (4) 67

Ans. (1)

Sol.
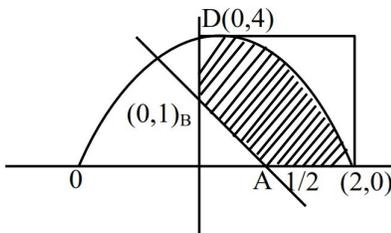
Required area $= \dfrac{2}{3} \times 8 - \dfrac{1}{2} \times \dfrac{1}{2} \times 1$

$= \dfrac{16}{3} - \dfrac{1}{4} = \dfrac{61}{12} = \dfrac{\alpha}{\beta}$

$\Rightarrow \alpha + \beta = 73$

12. Let $a_1, \dfrac{a_2}{2}, \dfrac{a_3}{2^2}, \ldots, \dfrac{a_{10}}{2^9}$ be a G.P. of common ratio $\dfrac{1}{\sqrt{2}}$. If $a_1 + a_2 + \ldots + a_{10} = 62$, then a_1 is equal to :

(1) $2\left(\sqrt{2} - 1\right)$ (2) $2 - \sqrt{2}$

(3) $\sqrt{2} - 1$ (4) $2(2 - \sqrt{2})$

Ans. (1)

Sol. $\dfrac{a_2}{2a_1} = \dfrac{a_3}{2a_2} = \dfrac{a_4}{2a_3} = \ldots = \dfrac{a_{10}}{2a_9} = \dfrac{1}{\sqrt{2}}$

$\therefore$ $a_1, a_2, a_3, \ldots, a_{10}$ are in G.P. with common ratio $\sqrt{2}$.

$\sum_{i=1}^{10} a_i = \dfrac{a_1\left(\left(\sqrt{2}\right)^{10} - 1\right)}{\sqrt{2} - 1} = 62$

$\Rightarrow a_1 = 2\left(\sqrt{2} - 1\right)$

13. Let $A = \{x : |x^2 - 10| \leq 6\}$ and $B = \{x : |x - 2| > 1\}$. Then

(1) $A \cup B = (-\infty, 1] \cup (2, \infty)$

(2) $A - B = [2, 3)$

(3) $A \cap B = [-4, -2] \cup [3, 4]$

(4) $B - A = (-\infty, -4) \cup (-2, 1) \cup (4, \infty)$

Ans. (4)

Sol. $|x^2 - 10| \leq 6$

$-6 \leq x^2 - 10 \leq 6$

$4 \leq x^2 \leq 16$

$A = [-4, -2] \cup [2, 4]$

$|x - 2| > 1$

$B = (-\infty, 1) \cup (3, \infty)$

$A \cup B = (-\infty, 1) \cup [2, \infty)$

$A \cap B = [-4, -2] \cup (3, 4]$

$A - B = [2, 3]$

$B - A = (-\infty, -4) \cup (-2, 1) \cup (4, \infty)$

14. For the matrices $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} -29 & 49 \\ -13 & 18 \end{bmatrix}$, if $(A^{15} + B)\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, then among the following which one is true?

(1) $x = 5, y = 7$ (2) $x = 18, y = 11$

(3) $x = 11, y = 2$ (4) $x = 16, y = 3$

Ans. (3)

Sol. Here $A^n = \begin{bmatrix} 2n+1 & -4n \\ n & -2n+1 \end{bmatrix}$

$\Rightarrow A^{15} = \begin{bmatrix} 31 & -60 \\ 15 & -29 \end{bmatrix}$

$\Rightarrow A^{15} + B = \begin{bmatrix} 2 & -11 \\ 2 & -11 \end{bmatrix}$

Now $\left(A^{15} + B\right)\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2 & -11 \\ 2 & -11 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow 2x - 11y = 0$

15. For a triangle ABC, let $\vec{p} = \overrightarrow{BC}, \vec{q} = \overrightarrow{CA}$ and $\vec{r} = \overrightarrow{BA}$. If $|\vec{p}| = 2\sqrt{3}, |\vec{q}| = 2$ and $\cos\theta = \dfrac{1}{\sqrt{3}}$, where θ is the angle between $\vec{P}$ and $\vec{q}$, then $\left|\vec{p} \times (\vec{q} - 3\vec{r})\right|^2 + 3|\vec{r}|^2$ is equal to:

(1) 340   (2) 220

(3) 410   (4) 200

Ans. (4)
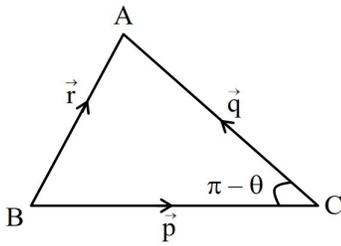
Sol.

$$\boxed{\vec{p} + \vec{q} = \vec{r}}$$

$$\cos(\pi - \theta) = \frac{|\vec{p}|^2 + |\vec{q}|^2 - |\vec{r}|^2}{2|\vec{p}||\vec{q}|}$$

$$\frac{-1}{\sqrt{3}} = \frac{12 + 4 - |\vec{r}|^2}{2 \cdot 2\sqrt{3} \cdot 2}$$

$$\boxed{|\vec{r}|^2 = 24}$$

$$\therefore \left|\vec{p} \times (\vec{q} - 3\vec{r})\right|^2 + 3|\vec{r}|^2$$

$$= \left|\vec{p} \times (\vec{q} - 3\vec{p} - 3\vec{q})\right|^2 + 72$$

$$= \left|\vec{p} \times (-3\vec{p} - 2\vec{q})\right|^2 + 72$$

$$= \left|-2\vec{p} \times \vec{q}\right|^2 + 72$$

$$= 4|\vec{p}|^2|\vec{q}|^2 \times \sin^2\theta + 72$$

$$= 4 \cdot 12 \cdot 4 \cdot \frac{2}{3} + 72$$

$$= 200$$

16. Let $y = y(x)$ be the solution of the differential equation $\sec x \dfrac{dy}{dx} - 2y = 2 + 3\sin x$, $x \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$, $y(0) = -\dfrac{7}{4}$. Then $y\left(\dfrac{\pi}{6}\right)$ is equal to:

(1) $-\dfrac{5}{2}$ (2) $-\dfrac{5}{4}$

(3) $-3\sqrt{3} - 7$ (4) $-3\sqrt{2} - 7$

Ans. (1)

Sol. $\dfrac{dy}{dx} - 2y\cos x = 2\cos x + 3\sin x.\cos x$

I.F. $= e^{-2\sin x}$

$$e^{-2\sin x}.y = \int e^{-2\sin x}(3\sin x\cos x + 2\cos x)dx$$

$$y.e^{-2\sin x} = e^{-2\sin x}\left(-\frac{3}{2}\sin x - \frac{7}{4}\right) + C$$

$$\Rightarrow y = -\frac{3}{2}\sin x - \frac{7}{4} + C.e^{2\sin x}$$

$$\because y(0) = -\frac{7}{4} \Rightarrow C = 0$$

$$y\left(\frac{\pi}{6}\right) = \frac{-3}{2} \cdot \frac{1}{2} - \frac{7}{4} = \frac{-5}{2}$$

17. Let $A = \{2, 3, 5, 7, 9\}$. Let R be the relation on A defined by x Ry if and only if $2x \leq 3y$. Let ℓ be the number of elements in R, and m be the minimum number of elements required to be added in R to make it a symmetric relation. Then $\ell + m$ is equal to:

(1) 23   (2) 25

(3) 21   (4) 27

Ans. (2)

Sol. $A = \{2,3,5,7,9\}$

$$y \geq \frac{2x}{3}$$

$$\left.\begin{array}{ll} x = 2, & y = 2,3,5,7,9 \\ x = 3, & y = 2,3,5,7,9 \\ x = 5, & y = 5,7,9 \\ x = 7, & y = 5,7,9 \\ x = 9 & y = 7,9 \end{array}\right\} \rightarrow \ell = 18$$

to make it symmetric elements to be added are
$\{(5,2), (7,2), (9,2), (5,3),(7,3),(9,3),(9,5)\}$

$$\boxed{m = 7}$$

$$\therefore \boxed{\ell + m = 25}$$

18. If the system of equations

$3x + y + 4z = 3$

$2x + \alpha y - z = -3$

$X + 2y + z = 4$

has no solution, then the value of α is equal to:

(1) 19   (2) 4

(3) 13   (4) 23

Ans. (1)

Sol. for no solution $\Delta = 0$

$$\begin{vmatrix} 3 & 1 & 4 \\ 2 & \alpha & -1 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow 3(a + 2) + 1(-1 - 2) + 4(4 - \alpha) = 0$

$\Rightarrow 19 - \alpha = 0 \Rightarrow \boxed{\alpha = 19}$

& for $\alpha = 19$

$$\Delta_x = \begin{vmatrix} 3 & 1 & 4 \\ -3 & 19 & -1 \\ 4 & 2 & 1 \end{vmatrix} = 3(21) + 1(-1) + 4(-82)$$

$\neq 0$

$\therefore$ no solution for $\alpha = 19$

19. Let z be the complex number satisfying $|z - 5| \le 3$ and having maximum positive principal argument.

Then $34\left|\dfrac{5z - 12}{5iz + 16}\right|^2$ is equal to:

(1) 16   (2) 12

(3) 26   (4) 20

Ans. (4)

Sol.

$|z - 5| \le 3$
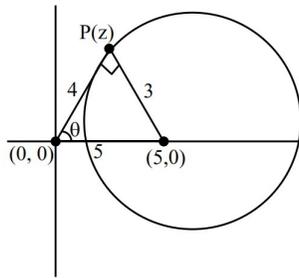
For arg(z) to be maximum, z lies at P.

$z \equiv (4\cos\theta, 4\sin\theta)$

$$\equiv \left(4 \cdot \left(\frac{4}{5}\right), 4\left(\frac{3}{5}\right)\right) = \left(\frac{16}{5}, \frac{12}{5}\right) = \frac{16}{5} + \frac{12i}{5}$$

Now, $34\left|\dfrac{5z - 12}{5iz + 16}\right|^2 = 34\left|\dfrac{(16 + 12i) - 12}{(16i - 12) + 16}\right|^2$

$$= 34\left|\frac{4 + 12i}{16i + 4}\right|^2$$

$$= 34\left(\frac{16 + 144}{256 + 16}\right) = 34\left(\frac{160}{272}\right) = 20$$

20. The largest $n \in N$, for which 7^n divides 101!, is :

(1) 16   (2) 18

(3) 15   (4) 19

Ans. (1)

Sol. Exponent of 7 in 101!

$$= \left[\frac{101}{7}\right] + \left[\frac{101}{7^2}\right] + \left[\frac{101}{7^3}\right] + \ldots\ldots$$

$= 14 + 2 = 16$

SECTION-B

21. Let $[\cdot]$ denote the greatest integer function and $f(x) = \lim\limits_{n \to \infty} \dfrac{1}{n^3} \sum\limits_{k=1}^{n}\left[\dfrac{k^2}{3^x}\right]$. Then $12\sum\limits_{j=1}^{\infty} f(j)$ is equal to _____.

Ans. (2)

Sol. $\sum\limits_{k=1}^{n}\left(\dfrac{k^2}{3^x} - 1\right) < \sum\limits_{k=1}^{n}\left[\dfrac{k^2}{3^x}\right] \le \sum\limits_{k=1}^{n}\dfrac{k^2}{3^x}$

$$\frac{n(n+1)(2n+1)}{6 \cdot 3^x} < \sum\limits_{k=1}^{n}\left[\frac{k^2}{3^x}\right] \le \frac{n(n+1)(2n+1)}{6 \cdot 3^x}$$

$$\lim\limits_{n \to \infty}\frac{n(n+1)(2n+1)}{6n^3 \cdot 3^x} < \lim\limits_{n \to \infty}\frac{1}{n^3}\sum\limits_{k=1}^{n}\left[\frac{k^2}{3^x}\right] \le \lim\limits_{n \to \infty}\frac{n(n+1)(2n+1)}{6 \cdot 3^x \cdot n^3}$$

$$\frac{1}{3^{x+1}} < \lim\limits_{n \to \infty}\frac{1}{n^3}\sum\limits_{k=1}^{n}\left[\frac{k^2}{3^x}\right] \le \frac{1}{3^{x+1}}$$

$\Rightarrow f(x) = \dfrac{1}{3^{x+1}}$

$\Rightarrow 12\sum\limits_{j=1}^{\infty}f(j) = 12\sum\limits_{j=1}^{\infty}\dfrac{1}{3^{j+1}} = 12\left[\dfrac{1}{9} + \dfrac{1}{27} + - - -\infty\right]$

$$= 12 \cdot \left(\frac{\frac{1}{9}}{1 - \frac{1}{3}}\right) = 2$$

22. If $\int_0^1 4\cot^{-1}(1-2x+4x^2)dx = a\tan^{-1}(2) - b\log_e(5)$, where $a, b \in \mathbf{N}$, then $(2a+b)$ is equal to _____.

Ans. (9)

Sol. Let $I = \int_0^1 \cot^{-1}(1-2x+4x^2)dx$

$I = \int_0^1 \left(\cot^{-1}(2x-1) - \cot^{-1}(2x)\right)dx$ …(1)

Applying king

$I = \int_0^1 \left(-\cot^{-1}(2x-1) + \cot^{-1}(2x-2)\right)dx$ …(2)

From (1) & (2)

$2I = \int_0^1 \left(\cot^{-1}(2x-2) - \cot^{-1}(2x)\right)dx$

$= \int_0^1 \cot^{-1}(2x-2)dx - \int_0^1 \cot^{-1}(2x)dx$

Applying King

$= \int_0^1 \cot^{-1}(-2x)dx - \int_0^1 \cot^{-1}(2x)dx$

$= \int_0^1 (\pi - \cot^{-1}(2x))dx - \int_0^1 \cot^{-1}(2x)dx$

$= \int_0^1 (\pi - 2\cot^{-1}(2x))dx$

$= \pi - 2\int_0^1 (\cot^{-1}2x)\cdot 1 dx$

By parts

$= \pi - 2\left[\left(x\cot^{-1}2x\right)_0^1 + \int_0^1 \frac{2x}{1+4x^2}dx\right]$

Let $1+4x^2 = t$

$8xdx = dt$

$= \pi - 2\left[\cot^{-1}2 + \frac{1}{4}\int_1^5 \frac{dt}{t}\right]$

$= \pi - 2\cot^{-1}2 - \frac{1}{2}\ell n5$

$2I = 2\tan^{-1}2 - \frac{1}{2}\ell n5$

$\Rightarrow 4I = 4\tan^{-1}2 - \ell n5$

$\therefore 2a + b = 8 + 1 = 9$

23. Let the maximum value of $(\sin^{-1}x)^2 + (\cos^{-1}x)^2$ for $x \in \left[-\frac{\sqrt{3}}{2}, \frac{1}{\sqrt{2}}\right]$ be $\frac{m}{n}\pi^2$, where $\gcd(m, n) = 1$.

Then $m + n$ is equal to _____.

Ans. (65)

Sol. $\left(\sin^{-1}x\right)^2 + \left(\cos^{-1}x\right)^2$

$= \left(\sin^{-1}x + \cos^{-1}x\right)^2 - 2\sin^{-1}x\cos^{-1}x$

$= \frac{\pi^2}{4} - 2\left(\sin^{-1}x\right)\left(\frac{\pi}{2} - \sin^{-1}x\right)$

$= 2\left(\sin^{-1}x - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{8}$ where $\sin^{-1}x \in \left[\frac{-\pi}{3}, \frac{\pi}{4}\right]$

Then max value occurs at $\sin^{-1}x = \frac{-\pi}{3}$

Which is $2\left(\frac{\pi}{3} + \frac{\pi}{4}\right)^2 + \frac{\pi^2}{8} = \frac{29\pi^2}{36}$

$\Rightarrow m = 29$ and $n = 36$

$\therefore m + n = 65$

24. If

$$\left(\frac{1}{^{15}C_0} + \frac{1}{^{15}C_1}\right)\left(\frac{1}{^{15}C_1} + \frac{1}{^{15}C_2}\right)\cdots\left(\frac{1}{^{15}C_{12}} + \frac{1}{^{15}C_{13}}\right)$$

$$= \frac{\alpha^{13}}{^{14}C_0\,^{14}C_1\cdots\,^{14}C_{12}}, \text{ then } 30\alpha \text{ is equal to } \_\_\_\_\_.$$

Ans. (32)

Sol. $\prod_{r=0}^{12}\left(\frac{1}{^{15}C_r} + \frac{1}{^{15}C_{r+1}}\right) = \prod_{r=0}^{12} \frac{\frac{16}{r+1}\cdot^{15}C_r}{^{15}C_r\cdot^{15}C_{r+1}}$

$= \prod_{r=0}^{12} \frac{16}{(r+1)\cdot\frac{15}{r+1}\cdot^{14}C_r} = \prod_{r=0}^{12} \frac{\left(\frac{16}{15}\right)}{^{14}C_r}$

$= \frac{\left(\frac{16}{15}\right)^{13}}{^{14}C_0\cdot^{14}C_1 \_\_\_\_\_ ^{14}C_{12}} \Rightarrow \alpha = \frac{16}{15}$

$\Rightarrow 30\alpha = 32$

25. If P is a point on the circle $x^2 + y^2 = 4$, Q is a point on the straight line $5x + y + 2 = 0$ and $x - y + 1 = 0$ is the perpendicular bisector of PQ, then 13 times the sum of abscissa of all such point P is _____.

Ans. (2)
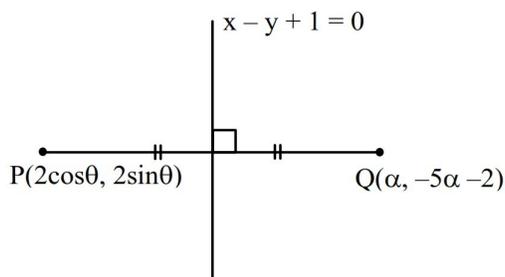
Mid point of PQ lies on $x - y + 1 = 0$

$$\frac{2\cos\theta + \alpha}{2} - \frac{2\sin\theta - 5\alpha - 2}{2} + 1 = 0$$

$2\cos\theta + \alpha - 2\sin\theta + 5\alpha + 2 + 2 = 0$

$\cos\theta - \sin\theta + 3\alpha + 2 = 0$ …(1)

$\because$ Slope of PQ is –1

$$\frac{2\sin\theta + 5\alpha + 2}{2\cos\theta - \alpha} = -1$$

$2\sin\theta + 5\alpha + 2 = -2\cos\theta + \alpha$

$\sin\theta + \cos\theta + 2\alpha + 1 = 0$ …(2)

eliminate α from (1) and (2)

$\Rightarrow \cos\theta + 5\sin\theta = 1$, $\theta \in [0, 2\pi]$

$\Rightarrow 5 \times 2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2} = 2\sin^2\dfrac{\theta}{2}$

$\therefore \sin\dfrac{\theta}{2} = 0 \Rightarrow \cos\theta = 1$

or

$\sin\dfrac{\theta}{2} = 5 \Rightarrow \cos\theta = -\dfrac{12}{13}$

Sum of all possible values of abscissa of point P is

$= 2 \times 1 + 2\left(\dfrac{-12}{13}\right) = \dfrac{2}{13}$

$\therefore$ 13 times sum of all possible values of abscissa of point P is 2.

PHYSICS

SECTION-A

26. Consider two identical metallic spheres of radius R each having charge Q and mass m. Their centers have an initial separation of $4R$. Both the spheres are given an initial speed of u towards each other. The minimum value of u, so that they can just touch each other is :

(Take $k = \dfrac{1}{4\pi \in_0}$ and assume $kQ^2 > Gm^2$ where G is the Gravitational constant)

(1) $\sqrt{\dfrac{kQ^2}{4mR}\left(1 - \dfrac{Gm^2}{kQ^2}\right)}$ (2) $\sqrt{\dfrac{kQ^2}{4mR}\left(1 + \dfrac{Gm^2}{kQ^2}\right)}$

(3) $\sqrt{\dfrac{kQ^2}{2mR}\left(1 - \dfrac{Gm^2}{kQ^2}\right)}$ (4) $\sqrt{\dfrac{kQ^2}{2mR}\left(1 - \dfrac{Gm^2}{2kQ^2}\right)}$

Ans. (1)

Sol. Using energy conservation

$(2)\left(\dfrac{1}{2}mu^2\right) - \dfrac{Gm^2}{4r} + \dfrac{KQ^2}{4r} = -\dfrac{Gm^2}{2r} + \dfrac{KQ^2}{2r}$

$u = \sqrt{\dfrac{1}{4mr}\left(KQ^2 - Gm^2\right)}$

27. The charge stored by the capacitor C in the given circuit in the steady state is ………. μC.
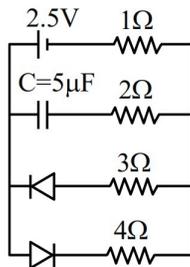
(1) 12.5 (2) 10

(3) 7.5 (4) 5

Ans. (2)

Sol.

in steady state
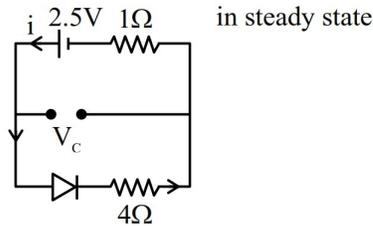
i = 2.5/5 = 0.5 A

$V_c = 4 \times 0.5$

$V_c = 2V$

charge

$Q = CV_c$

$= 5 \times 2$

$= 10 \ \mu C$

28. The total length of potentiometer wire AB is 50 cm in the arrangement as shown in figure. If P is the point where the galvanometer shows zero reading then the length AP is ………. cm.
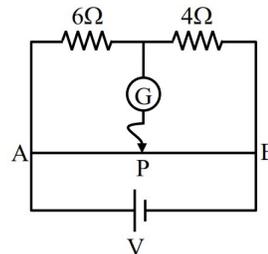
(1) 15 (2) 30

(3) 25 (4) 20

Ans. (2)

Sol. $\dfrac{6}{R_{AP}} = \dfrac{4}{R_{PB}}$;

$\ell_{AP} + \ell_{PB} = 50$ ….(i)

$\dfrac{R_{AP}}{R_{PB}} = \dfrac{\ell_{AP}}{\ell_{PB}} = \dfrac{3}{2}$

$\ell_{AP} = \dfrac{3}{5} \times 50 = 30 \ cm$

29. A capacitor C is first charged fully with potential difference of V_0 and disconnected from the battery. The charged capacitor is connected across an inductor having inductance L. In t s 25% of the initial energy in the capacitor is transferred to the inductor. The value of t is_____ s.

(1) $\dfrac{\pi\sqrt{LC}}{3}$ 　　　　　 (2) $\dfrac{\pi\sqrt{LC}}{6}$

(3) $\dfrac{\pi\sqrt{LC}}{2}$ 　　　　　 (4) $\pi\sqrt{\dfrac{LC}{2}}$

Ans. (2)

Sol. $U_{c_f} = 75\% U_{c_i}$

$$Q_F^2 = \frac{3}{4}Q_I^2$$

$$Q_i\cos\omega t = \frac{\sqrt{3}}{2}Q_i \Rightarrow t = \frac{T}{12}$$

$$t = \frac{\pi}{6}\sqrt{LC}$$

30. The r.m.s speed of oxygen molecules at 47°C is equal to that of the hydrogen molecules kept at_____°C. (Mass of oxygen molecule/mass of hydrogen molecule=32/2)

(1) –235 　　　　　 (2) –100

(3) –253 　　　　　 (4) –20

Ans. (3)

Sol. $V_{rms} = \sqrt{\dfrac{3RT}{M}}$

$$V_{rmsO_2} = V_{rmsH_2}$$

$$T_{O_2} = 273 + 47 = 320K$$

$$\sqrt{\frac{3RT_{O_2}}{M_{O_2}}} = \sqrt{\frac{3R\,T_{H_2}}{M_{H_2}}}$$

$$\frac{T_2}{M_{O_2}} = \frac{T_{H_2}}{M_{H_2}}$$

$$\frac{320}{32} = \frac{T_{H_2}}{2}$$

$$T_{H_2} = 20K$$

$$T_{H_2} = -253°C$$

31. Two cars A and B each of mass 10^3 kg are moving on parallel tracks separated by a distance of 10 m, in same direction with speeds 72 km/h and 36 km/h. The magnitude of angular momentum of car A with respect to car B is ___ J.s.

(1) 3.6×10^5 　　　　　 (2) 10^5

(3) 3×10^5 　　　　　 (4) 2×10^5

Ans. (2)

Sol. $L = m.V_{rel}\, r_\perp$

$$= 1000 \times \left(36 \times \frac{5}{18}\right) \times 10$$

$$= 10^5 \text{ kg m}^2/\text{s}$$

32. The pulley shown in figure is made using a thin rim and two rods of length equal to diameter of the rim. The rim and each rod have a mass of M. Two blocks of mass of M and m are attached to two ends of a light string passing over the pulley, which is hinged to rotate freely in vertical plane about its centre. The magnitudes of the acceleration experienced by the blocks is____ (assume no slipping of string on pulley.)
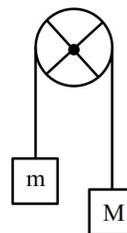
(1) $\dfrac{(M-m)g}{\left[\left(\dfrac{13}{6}\right)M + m\right]}$

(2) $\dfrac{(M-m)g}{M+m}$

(3) $\dfrac{(M-m)g}{\left[\left(\dfrac{8}{3}\right)M + m\right]}$

(4) $\dfrac{(M-m)g}{2M+m}$

Ans. (3)

Sol.
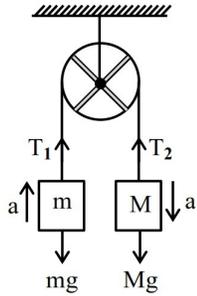
$Mg - T_2 = Ma$ ...(1)

$T_1 - mg = ma$ ...(2)

$(T_2 - T_1)r = I\dfrac{a}{r}$ ...(3)

(1) + (2) + (3)

$(M - m)g = \left(M + m + \dfrac{I}{r^2}\right)a$

Here $I = Mr^2 + \dfrac{M \times (2r)^2}{12} \times 2$

$= \left(1 + \dfrac{2}{3}\right)Mr^2$

$= \dfrac{5}{3}Mr^2$

$(M - m)g = \left[M + m + \dfrac{5M}{3}\right]a$

$a = \dfrac{(M - m)g}{\left[M + m + \dfrac{5M}{3}\right]}$

33. The kinetic energy of a simple harmonic oscillator is oscillating with angular frequency of 176 rad/s. The frequency of this simple harmonic oscillator is_____Hz. $\left[\text{take } \pi = \dfrac{22}{7}\right]$

(1) 14   (2) 88

(3) 28   (4) 176

Ans. (1)

Sol. $\omega = 176$ rad/sec

$f_k = \dfrac{\omega}{2\pi} = \dfrac{176}{2 \times 22} \times 7$

$= \dfrac{176}{44} \times 7$

$= 4 \times 7 = 28$ Hz

So frequency of oscillator

$= \dfrac{f_k}{2} = 14$ Hz

34. Given below are two statements:

Statement I : In a Young's double slit experiment, the angular separation of fringes will increase as the screen is moved away from the plane of the slits

Statement II : In a Young's double slit experiment, the angular separation of fringes will increase when monochromatic source is replaced by another monochromatic source of higher wavelength

In the light of the above statements, choose the **correct answer** from the options given below:

(1) Both **Statement I** and **Statement II** are true

(2) Both **Statement I** and **Statement II** are false

(3) **Statement I** is false but **Statement II** is true

(4) **Statement I** is true but **Statement II** is false

Ans. (3)

Sol. Angular fringe width $= \dfrac{\lambda}{d}$

35. A battery with EMF E and internal resistance r is connected across a resistance R. The power consumption in R will be maximum when :

(1) $R = 2r$ (2) $R = \dfrac{r}{2}$

(3) $R = \sqrt{2}r$ (4) $R = r$

Ans. (4)

Sol. For maximum power drawn across load

Resistance $R_{Load} = R_{internal}$

$\boxed{R = r}$

36. Keeping the significant figures in view, the sum of the physical quantities 52.01m, 153.2 m and 0.123 m is :

(1) 205 m   (2) 205.333 m

(3) 205.33 m   (4) 205.3 m

Ans. (4)

Sol. L = 52.01 + 153.2 + 0.123

$= 205.333$

$= 205.3$

37. A spherical body of radius r and density σ falls freely through a viscous liquid having density ρ and viscosity η and attains a terminal velocity v_0. Estimated maximum error in the quantity η is : (Ignore errors associated with σ, ρ and g, gravitational acceleration)

(1) $2\dfrac{\Delta r}{r} - \dfrac{\Delta v_0}{v_0}$

(2) $\dfrac{2\Delta r}{r} + \dfrac{\Delta v_0}{v_0}$

(3) $2\left[\dfrac{\Delta r}{r} + \dfrac{\Delta v_0}{v_0}\right]$

(4) $2\left[\dfrac{\Delta r}{r} - \dfrac{\Delta v_0}{v_0}\right]$

Ans. (2)

Sol. $v_0 = \dfrac{2}{9}\dfrac{r^2 g}{n}(\rho_B - \rho_L)$

$n = \dfrac{2}{9}\dfrac{r^2 g}{v_0}(\rho_B - \rho_L)$

$\dfrac{\Delta n}{n} = \dfrac{2\Delta r}{r} + \dfrac{\Delta v_0}{v_0}$

38. Surface tension of two liquids (having same densities), T_1 and T_2, are measured using capillary rise method utilizing two tubes with inner radii of r_1 and r_2 where $r_1 > r_2$. The measured liquid heights in these tubes are h_1 and h_2 respectively. [Ignore the weight of the liquid about the lowest point of miniscus]. The heights h_1 and h_2 and surface tensions T_1 and T_2 satisfy the relation :

(1) $h_1 < h_2$ and $T_1 = T_2$

(2) $h_1 = h_2$ and $T_1 = T_2$

(3) $h_1 > h_2$ and $T_1 = T_2$

(4) $h_1 > h_2$ and $T_1 < T_2$

Ans. (1)

Sol. $h = \dfrac{2T}{\rho g r}$

$h \propto \dfrac{1}{r}$

If $r_1 > r_2 \Rightarrow h_2 > h_1$

39. A river of width 200 m is flowing from west to east with a speed of 18 km/h. A boat, moving with speed of 36 km/h in still water, is made to travel one-round trip (bank to bank of the river). Minimum time taken by the boat for this journey and also the displacement along the river bank are _____ and _____ respectively.

(1) 20 s and 100 m

(2) 40 s and 0 m

(3) 40 s and 200 m

(4) 40 s and 100 m

Ans. (3)

Sol.

Minimum time :

$t_{min} = \dfrac{200}{10} = 20\,\sec$
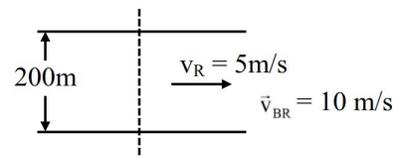
For round trip = 40 sec.

Displacement along river bank = $40 \times 5 = 200$ m

40. Two known resistance of R Ω and 2R Ω and and one unknown resistance X Ω are connected in a circuit as shown in the figure. If the equivalent resistance between points A and B in the circuit is X Ω, then the value of X is _____ Ω.
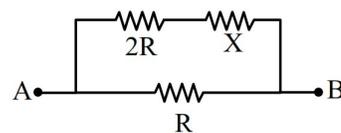
(1) $\left(\sqrt{3} - 1\right)R$

(2) R

(3) $2\left(\sqrt{3} - 1\right)R$

(4) $\left(\sqrt{3} + 1\right)R$

Ans. (1)

Sol. $\dfrac{(2R + x).(R)}{3R + x} = x$

$x^2 + 2Rx - 2R^2 = 0$

$x = (\sqrt{3} - 1)R$

41. The energy of an electron in an orbit of the Bohr's atom is $-0.04E_0$ eV where E_0 is the ground state energy. If L is the angular momentum of the electron in this orbit and h is the Planck's constant, then $\dfrac{2\pi L}{h}$ is _____ :

(1) 2 (2) 4

(3) 5 (4) 6
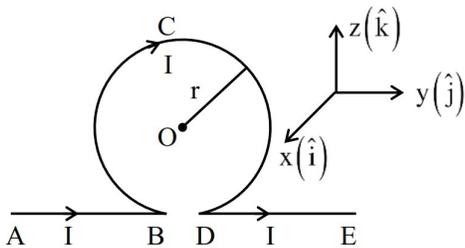
Ans. (3)

Sol. Angular momentum $L = \dfrac{nh}{2\pi}$

$n = \dfrac{2\pi L}{h}$

Energy $E = -\dfrac{13.6}{n^2} \cdot z^2$

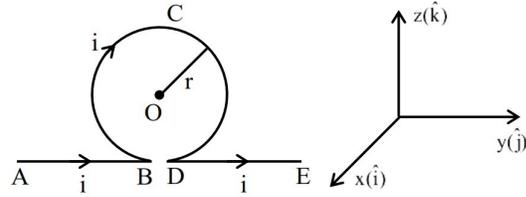$E \Rightarrow -\dfrac{E_0}{n^2} = -0.04E_0$

$n^2 = 25,\ n = 5$

42. An infinitely long straight wire carrying current I is bent in a planer shape as shown in the diagram. The radius of the circular part is r. The magnetic field at the centre O of the circular loop is :

(1) $\dfrac{\mu_0}{2\pi}\dfrac{I}{r}(\pi+1)\hat{i}$ (2) $-\dfrac{\mu_0}{2\pi}\dfrac{I}{r}(\pi-1)\hat{i}$

(3) $\dfrac{\mu_0}{2\pi}\dfrac{I}{r}(\pi-1)\hat{i}$ (4) $-\dfrac{\mu_0}{2\pi}\dfrac{I}{r}(\pi+1)\hat{i}$

Ans. (2)

Sol.

$\vec{B}_0 = \vec{B}_{AB} + \vec{B}_{DE} + \vec{B}_{BCD}$

$= \dfrac{\mu_0 i}{4\pi r}\hat{i} + \dfrac{\mu_0 i}{4\pi r}\hat{i} - \dfrac{\mu_0 i}{2r}\hat{i}$

$= \dfrac{\mu_0 i}{2\pi r}\hat{i} - \dfrac{\mu_0 i}{2r}\hat{i}$

$= \dfrac{\mu_0 i}{2\pi r}(1-\pi)\hat{i}$

$= -\dfrac{\mu_0 i}{2\pi r}(\pi-1)\hat{i}$

43. A large drum having radius R is spinning around its axis with angular velocity ω, as shown in figure. The minimum value of ω so that a body of mass M remains stuck to the inner wall of the drum, taking the coefficient of friction between the drum surface and mass M is μ, is :
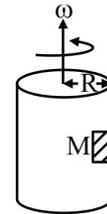
(1) $\sqrt{\dfrac{\mu g}{R}}$ (2) $\sqrt{\dfrac{2g}{\mu R}}$

(3) $\sqrt{\dfrac{g}{2\mu R}}$ (4) $\sqrt{\dfrac{g}{\mu R}}$

Ans. (4)
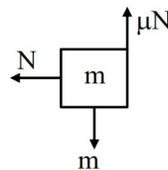
Sol.

$N = m\omega^2 r$, $mg = \mu N$

$\mu \times m\omega^2 r = mg$

$\omega = \sqrt{\dfrac{g}{\mu r}}$

44. A body of mass 2 kg is moving along x-direction such that its displacement as function of time is given by $x(t) = \alpha t^2 + \beta t + \gamma$m, where $\alpha = 1$ m/s^2, $\beta = 1$ m/s and $\gamma = 1$m. The work done on the body during the time interval t = 2 s to t = 3 s, is _____ J.

(1) 49   (2) 42

(3) 24   (4) 12

Ans. **(3)**

Sol. $x(t) = t^2 + t + 1$

$v(t) = 2t + 1$

$a(t) = 2$

$F = 4$N.

Displacement = $x(3) - x(2)$

$= 13 - 7 = 6$M

$W = F.S = 4 \times 6 = 24$ J

45. As shown in the diagram, when the incident ray is parallel to base of the prism, the emergent ray grazes along the second surface.
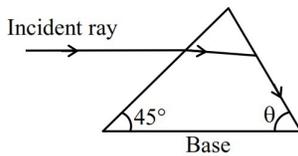
If refractive index of the material of prism is $\sqrt{2}$, the angle θ of prism is.

(1) 60°   (2) 75°

(3) 90°   (4) 45°

Ans. **(1)**

Sol.
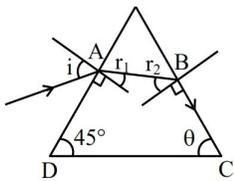
For grazing emergence

$\sin r_2 = \dfrac{1}{\mu}$

By Snell's Law at incident surface

$1 \times \dfrac{1}{\sqrt{2}} = \sqrt{2}\sin r_1$

$r_1 = 30$

$r_1 + r_2 = A$

$A = 75$

$75 + 45 + \theta = 180°$

$\theta = 60°$

SECTION-B

46. An electromagnetic wave of frequency 100 MHz propagates through a medium of conductivity, σ = 10 mho/m. The ratio of maximum conducting current density to maximum displacement current density is _____.

$$\left[\text{Take } \frac{1}{4\pi \epsilon_0} = 9 \times 10^9 \, \text{Nm}^2 / \text{C}^2 \right]$$

Ans. **(1800)**

Sol. A

$j_c = \sigma E$

$E \Rightarrow E_0 \sin(\omega t - kx)$

$j_c = \sigma E_0 \sin(\omega t - kx)$

$\Rightarrow (j_c)_{max} = \sigma E_0$ ....(i)

$J_d \Rightarrow \dfrac{i_d}{A} = \dfrac{1}{A} \times \varepsilon_0 \dfrac{AdE}{dt}$

$\Rightarrow \varepsilon_0 \times E_0 \omega \cos(\omega t - kx)$

$(j_d)_{max} \Rightarrow \varepsilon_0 E_0 \omega$ ....(ii)

(i)/(ii)

$\dfrac{(j_c)_{max}}{(j_d)_{max}} = \dfrac{\sigma E_0}{\varepsilon_0 \omega E_0} \Rightarrow \dfrac{\sigma}{\varepsilon_0 \omega}$

$\Rightarrow \dfrac{10 \times 4\pi \times 9 \times 10^9}{2\pi \times 100 \times 10^6}$

$\Rightarrow 1800$

47. The terminal velocity of a metallic ball of radius 6 mm in a viscous fluid is 20 cm/s. The terminal velocity of another ball of same material and having radius 3 mm in the same fluid will be _____ cm/s.

Ans. **(5)**

Sol. We know :

Terminal velocity $\propto$ (radius)^2

$\dfrac{(v_T)_1}{(v_T)_2} = \left(\dfrac{6}{3}\right)^2$

$(v_T)_2 = \dfrac{(v_T)_1}{4} = 5$ cm/sec

48. A particle having electric charge 3×10^{-19} C and mass 6×10^{-27} kg is accelerated by applying an electric potential of 1.21 V. Wavelength of the matter wave associated with the particle is $\alpha \times 10^{-12}$ m. The value of α is _____.
(Take Planck's constant = 6.6×10^{-34} J.s)

Ans. (10)

Sol. $\lambda = \dfrac{h}{\sqrt{2mqV}}$

$\lambda = \dfrac{6.6 \times 10^{-34}}{\sqrt{2 \times 18 \times 10^{-46} \times 1.21}}$

$\lambda = 10^{-11}$ m $= 10 \times 10^{-12}$ m

$\alpha = 10$

49. In a Young's double slit experiment set up, the two slits are kept 0.4. mm apart and screen is placed at 1 m from slits. If a thin transparent sheet of thickness 20 μm is introduced in front of one of the slits then centre bring fringe shifts by 20 mm on the screen. The refractive index of transparent sheet is given by $\dfrac{\alpha}{10}$, where α is _____.

Ans. (14)

Sol. $y_{shift} = \dfrac{(\mu - 1)t.D}{d}$

$20910^{-3} = \dfrac{(\mu - 1) \times 20 \times 10^{-6} \times 1}{0.4 \times 10^{-3}}$

$(\mu - 1) \Rightarrow 0.4$

$\mu \Rightarrow 1.4$

$\dfrac{\alpha}{10} = 1.4$, $\alpha = 14$

50. A diatomic gas ($\gamma = 1.4$) does 100 J of work when it is expanded isobarically. Then the heat given to the gas _____ J.

Ans. (350)

Sol. w = 100 J = nRΔT for isobaric process.

$Q = nC_p\Delta T = \left(\dfrac{f}{2} + 1\right)nR\Delta T$

$= \dfrac{7}{2}.(100) = 350$ Joule.

CHEMISTRY

51. Consider the following spectral lines for atomic hydrogen:

A. First line of Paschen series

B. Second line of Balmer series

C. Third line of Paschen series

D. Fourth line of Bracket sereis

The correct arrangement of the above lines in ascending order of energy is:

(1) $D < C < A < B$

(2) $A < B < C < D$

(3) $C < D < B < A$

(4) $D < A < C < B$

Ans. (4)

Sol. $\Delta E = 13.6\, Z^2 \left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2} \right)$

Series	n_1	n_2
(A) Paschen (1st line)	3	4
(B) Balmer (2nd line)	2	4
(C) Paschen (3rd line)	3	6
(D) Bracket (4th line)	4	8

So correct ascending order of energy of above lines is :

$D < A < C < B$

52. Match **List-I** with **List-II**.

List-I		List-II
Pair of Compounds		**Type of Isomers**
A.	2-Methylpropene and but-1-ene	I. Stereoisomers
B.	Cis-but-2-ene and trans-but-2-ene	II. Position isomers
C.	2-Butanol and diethyl ether	III. Chain isomers
D.	But-1-ene and but-2-ene	IV. Functional group isomers

Choose the **correct** answer from the options given below:

(1) A-III, B-I, C-IV, D-II

(2) A-III, B-I, C-II, D-IV

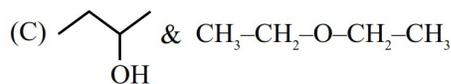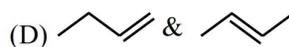(3) A-I, B-IV, C-III, D-II

(4) A-II, B-I, C-IV, D-III

Ans. (2)
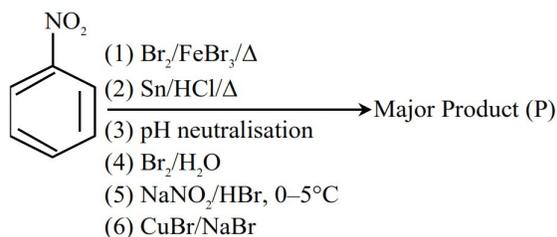
Sol.

(A) &

(III) Chain isomer

(B) &

(I) Streoisomers

(C) & $CH_3–CH_2–O–CH_2–CH_3$

(IV) Functional isomers

(D) &

(IV) Positional isomers

53.

(1) $Br_2/FeBr_3/\Delta$

(2) $Sn/HCl/\Delta$

(3) pH neutralisation

(4) Br_2/H_2O

(5) $NaNO_2/HBr$, 0–5°C

(6) $CuBr/NaBr$

$\longrightarrow$ Major Product (P)

Consider the above sequence of reactions. The number of bromine atom(s) in the final product (P) will be:

(1) 1   (2) 6

(3) 5   (4) 3

Ans. (3)

Sol.

Number of Br atom in major product (P) = 5

54. Aqueous HCl reacts with $MnO_2(s)$ to form $MnCl_2(aq)$, $Cl_2(g)$ and $H_2O(l)$. What is the weight (in g) of Cl_2 liberated when 8.7 g of $MnO_2(s)$ is reacted with excess aqueous HCl solution ? (Given Molar mass in g mol^{-1} Mn = 55, Cl = 35.5, O = 16, H = 1)

(1) 7.1   (2) 71

(3) 21.3   (4) 14.2

Ans. (1)

Sol. $MnO_2 + 4HCl \rightarrow MnCl_2 + Cl_2 + 2H_2O$

$\dfrac{8.7}{87}$ Excess

= 0.1 mole   0.1 mole

Wt. of Cl_2 obtained = 0.1 × 71 = 7.1 g

55. By usual analysis, 1.00g of compound (X) gave 1.79g of magnesium pyrophosphate. The percentage of phosphorus in compound (X) is: (nearest integer)

(Given, molar mass in g mol^{-1}: O = 16, Mg = 24, P = 31)

(1) 50   (2) 30

(3) 20   (4) 40

Ans. (1)

Sol. % of P = $\dfrac{n_{Mg_2P_2O_7} \times 2 \times 31}{w_{(unknown\ compound)}} \times 100$

$= \dfrac{\left(\dfrac{1.79}{222} \times 2 \times 31\right)}{1} \times 100$

$= 49.99\% \approx 50\%.$

56. Consider the following data:

$\Delta_f H^{\ominus}$(methane, g) = $-X$ kJ mol^{-1}

Enthalpy of sublimation of graphite = Y kJ mol^{-1}

Dissociation enthalpy of H_2 = Z kJ mol^{-1}

The bond enthalpy of C – H bond is given by:

(1) $\dfrac{X + Y + 2Z}{4}$

(2) $\dfrac{X + Y + 4Z}{2}$

(3) $X + Y + Z$

(4) $\dfrac{-X + Y + Z}{4}$

Ans. (1)

Sol. $C(s) + 2H_2(g) \rightarrow CH_4(g)$

$-x = (\Delta H_{sub}$ of carbon$) + 2 \times ($B.E. of H – H$)$

$-4 \times ($B.E. of C – H$)$

$-x = y + 2z - 4 ($B.E. of C – H$)$

B.E. of C – H = $\dfrac{y + 2z + x}{4}$.

57. Match **List-I** with **List-II**.

List-I		List-II
Reagents		**Reaction Name (Involving aldehydes)**
A. H_2, Pd-BaSO$_4$	I.	Etard Reaction
B. $SnCl_2$, HCl	II.	Rosenmund Reduction
C. CrO_2Cl_2, CS_2	III.	Gatterman – Koch Reaction
D. CO, HCl, Anhyd. AlCl$_3$	IV.	Stephen Reaction

Choose the **correct** answer from the options given below:

(1) A-II, B-III, C-IV, D-I

(2) A-IV, B-III, C-I, D-II

(3) A-IV, B-I, C-II, D-III

(4) A-II, B-IV, C-I, D-III

Ans. (4)

Sol. NCERT Name reaction theory based

58. Decomposition of A is a first order reaction at T(K) and is given by $A(g) \rightarrow B(g) + C(g)$. In a closed 1 L vessel, 1 bar A(g) is allowed to decompose at T(K). After 100 minutes, the total pressure was 1.5 bar. What is the rate constant (in min^{-1}) of the reaction ? (log 2 = 0.3)

(1) 6.9×10^{-1}

(2) 6.9×10^{-3}

(3) 6.9×10^{-2}

(4) 6.9×10^{-4}

Ans. (2)

Sol.
$$A_g \rightarrow B_g + C_g$$

$$1 \quad\quad - \quad\quad -$$

$$1-P \quad P \quad\quad P$$

$$P_{total} = 1 + P$$

$$1.5 = 1 + P$$

$$P = 0.5$$

$$K = \frac{1}{100} \ln \frac{1}{0.5}$$

$$= \frac{0.693}{100}$$

$$= 6.9 \times 10^{-3} \text{ min}^{-1}$$

59. The correct order of reactivity of the following benzyl halides towards reaction with KCN is:
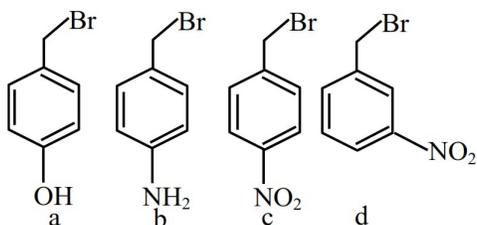
(1) $a > b > c > d$

(2) $b > a > d > c$

(3) $b > a > c > d$

(4) $a > b > d > d$

Ans. (2)

Sol. This is S_N1 reaction.

Rate of S_N1 reaction $\propto$ stability of carbocation

60. Given below are two statements:

Statement-I: The correct order in terms of atomic/ionic radii is $Al > Mg > Mg^{2+} > Al^{3+}$.

Statement-II: The correct order in terms of the magnitude of electron gain enthalpy is $Cl > Br > S > O$.

In the light of the above statements, choose the **correct** answer from the options given below:

(1) Both **Statement I** and **Statement II** are false

(2) **Statement I** is false but **Statement II** is true

(3) **Statement I** is true but **Statement II** is false

(4) Both **Statement I** and **Statement II** are true

Ans. (2)

Sol. Correct order of size is $Mg > Al > Mg^{2+} > Al^{3+}$

Atomic size depends mainly upon $Z_{effective}$ and shell number.

Generally on moving down the group electron affinity decreases and on moving across the period electron affinity increase.

In the periodic table Cl has maximum electron affinity. Halogen has higher electron affinity than Chalcogen.

$Cl > Br > S > O$

61. The **correct** statements are:

A. Activation energy for enzyme catalysed hydrolysis of sucrose is lower than that of acid catalysed hydrolysis.

B. During denaturation, secondary and tertiary structures of a protein are destroyed but primary structure remains intact.

C. Nucleotides are joined together by glycosidic linkage between C_1 and C_4 carbons of the pentose sugar

D. Quaternary structure of proteins represents overall folding of the polypeptide chain.

Choose the **correct** answer from the options given below:

(1) A, C and D Only

(2) A, B and D Only

(3) A and B Only

(4) B and C Only

Ans. (3)

Sol. Activation energy for enzyme catalysed hydrolysis of sucrose is lower than that of acid catalysed hydrolysis.
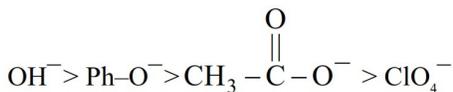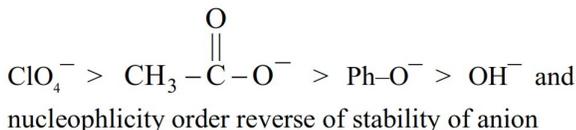
During denaturation secondary and tertiary structures of a protein are destroyed but primary structure remains intact.

62. The correct order of the rate of the reaction for the following reaction with respect to nucleophiles is:

$$CH_3Br + Nu^{\ominus} \longrightarrow CH_3Nu + Br^{\ominus}$$

(1) $PhO^- >^- OH > CH_3COO^- > ClO_4^-$

(2) $ClO_4^- > CH_3COO^- > {}^-OH > PhO^-$

(3) $CH_3COO^- > PhO^- > {}^-OH > ClO_4^-$

(4) $^-OH > PhO^- > CH_3COO^- > ClO_4^-$
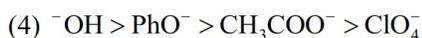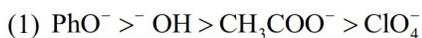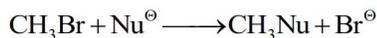
Ans. (4)

Sol. Stability order of anion

$$ClO_4^- > CH_3-\overset{\overset{\displaystyle O}{\|}}{C}-O^- > Ph-O^- > OH^-$$ and

nucleophlicity order reverse of stability of anion

$$OH^- > Ph-O^- > CH_3-\overset{\overset{\displaystyle O}{\|}}{C}-O^- > ClO_4^-$$

63. Given below are two statements:

Statement I: Crystal Field Stabilization Energy (CFSE) of $[Cr(H_2O)_6]^{2+}$ is greater than that of $[Mn(H_2O)_6]^{2+}$.

Statement II: Potassium ferricyanide has a greater spin-only magnetic moment than sodium Ferrocyanide.

In the light of the above statements, choose the **correct** answer from the options given below:

(1) Both **Statement I** and **Statement II** are true

(2) Both **Statement I** and **Statement II** are false

(3) **Statement I** is true but **Statement II** is false

(4) **Statement I** is false but **Statement II** is true

Ans. (1)

Sol. $[Mn(H_2O)_6]^{2+} \Rightarrow$ CFSE value is zero because of d^5 configuraiton with WFL in coordination number 6 $[Cr(H_2O)_6]^{2+}$.

$[Cr(H_2O)_6]^{2+} \Rightarrow$ CFSE value is $-0.6\Delta_0$ because of d^4 configuraiton with WFL in coordination number 6.

For : $K_3[Fe(CN)_6]$, $\mu = \sqrt{1(1+2)} = \sqrt{3}$ B.M.

For : $Na_4[Fe(CN)_6]$, $\mu = \sqrt{0}$ B.M.

64. The correct increasing order of C – H(A), C – O(B), C = O(C) and C ≡ N(D) bonds in terms of covalent bond length is:

(1) A < B < C < D

(2) A < D < C < B

(3) D < C < B < A

(4) D < C < A < B

Ans. (2)

Sol. C–H (A)   107 pm

C≡N (D)   116 pm

C–O (B)   143 pm

C=O (C)   121 pm

65. Given below are four compounds:
(a) n-propyl choride
(b) iso-propyl chloride
(c) sec-butyl chloride
(d) neo-pentyl chloride

Percentage of carbon in the one which exhibits optical isomerism is:

(1) 52   (2) 56   (3) 46   (4) 40

Ans. (1)

Sol. 2-Chlorobutane is optically active and chiral molecule

Molecular formula $\Rightarrow C_4H_9Cl$

Molar mass $= 48 + 9 + 35.5 = 92.5$

$\%OC = \dfrac{48}{92.5} \times 100 = 51.89\%$

66. Given below are some of the statements about Mn and Mn_2O_7. Identify the correct statements

A. Mn forms the oxide Mn_2O_7 in which Mn is in its highest oxidation state.

B. Oxygen stabilizes the Mn in higher oxidation states by forming multiple bonds with Mn

C. Mn_2O_7 is an ionic oxide.

D. The structure of Mn_2O_7 consists of one bridged oxygen.

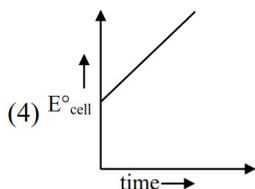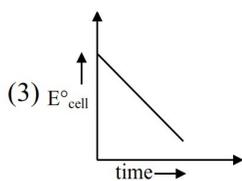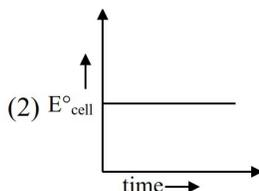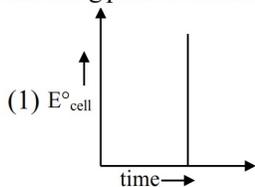Choose the **correct** answer from the options given below:

(1) A, B, C and D
(2) A, B and D Only
(3) A, C and D Only
(4) A, B and C Only

Ans. (2)

Sol. Mn_2O_7 : Mn in +7 oxidation state.
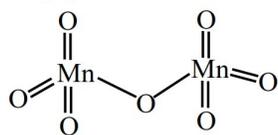
67. For a closed circuit Daniell cell, which of the following plots is the accurate one at a given temperature ?

(1) E°_{cell}

time→

(2) E°_{cell}

time→

(3) E°_{cell}

time→

(4) E°_{cell}

time→

Ans. (2)

Sol. E^0_{cell} remain constant with time.

68. Given below are two statements :

Statement – I : Compound (X), shown below , dissolves in $NaHCO_3$ solution and has two chiral carbon atoms

(X)
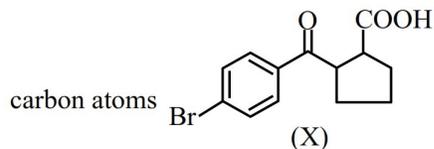
Statement – II : Compound (Y), shown below, has two carbons with sp^3 hybridization, one carbon with sp^2 and one carbon with sp hybridization
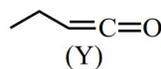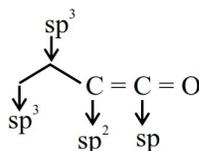

(Y)

In the light of the above statements, choose the **correct** answer from the options given below :

(1) **Statement I** is true but **Statement II** is false

(2) **Statement I** is false but **Statement II** is true

(3) Both **Statement I** and **Statement II** are true

(4) Both **Statement I** and **Statement II** are false

Ans. (3)

Sol.
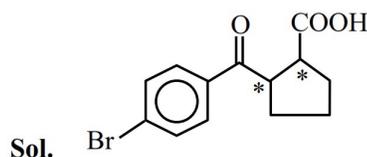
Two chiral centre and due to presence of –COOH compound dissolves in $NaHCO_3$.

69. Given below are two statements :

Statement I : The correct order in terms of bond dissociation enthalpy is $Cl_2 > Br_2 > F_2 > I_2$

Statement II : The correct trend in the covalent character of the metal halides is $[SnCl_4 > SnCl_2]$, $[PbCl_4 > PbCl_2]$ and $[UF_4 > UF_6]$

In the light of the above statements, choose the **correct** answer from the options given below :

(1) **Statement I** is true but **Statement II** is false

(2) Both **Statement I** and **Statement II** are true

(3) **Statement I** is false but **Statement II** is true

(4) Both **Statement I** and **Statement II** are false

Ans. (1)

Sol. Statement-I :

Bond energy order is $Cl_2 > Br_2 > F_2 > I_2$

Bond energy increases with increase in bond order.

Statement-II

Correct order of covalent character

According to the Fajan's rule, higher the charge on cation , greater is the covalent character.

$PbCl_2 < PbCl_4$,

$UF_6 > UF_4$,

$SnCl_4 > SnCl_2$

70. On heating a mixture of common salt and $K_2Cr_2O_7$ in equal amount along with concentrated H_2SO_4 in a test tube, a gas is evolved. Formula of the gas evolved and oxidation state of the central metal atom in the gas respectively are :

(1) CrO_2Cl_2 and +5   (2) CrO_2Cl_2 and +6

(3) $Cr_2O_2Cl_2$ and +6   (4) $Cr_2O_2Cl_2$ and +3

Ans. (2)

Sol. $4NaCl + K_2Cr_2O_7 + 6H_2SO_4$

$\longrightarrow 2KHSO_4 + 2CrO_2Cl_2 + 4NaHSO_4 + 3H_2O$
(Chromyl
chloride)

In Chromyl chloride Cr is in +6 oxidation state.

SECTION-B

71. The first and second ionization constants of H_2X are 2.5×10^{-8} and 1.0×10^{-13} respectively. The concentration of X^{2-} in 0.1 M H_2X solution is _____ $\times 10^{-15}$ M. (Nearest Integer)

Ans. (100)

Sol. $H_2X \rightleftharpoons H^+ + HX^-,$

    0.1–x   x +y   x–y

$2.5 \times 10^{-8} = \dfrac{(x+y)(x-y)}{0.1-x}$

$HX^- \rightleftharpoons H^+ + X^{2-},$

    x–y   x +y   y

$1 \times 10^{-13} = \dfrac{(x+y)(y)}{x-y}$

Approximate : $K_{a_1} >> K_{a_2} \Rightarrow$ So $x >> y$.

$x + y \approx x, x - y \approx x$

$10^{-13} = \dfrac{x.y}{x}$

$y = 10^{-13}$

$[X^{2-}] = 10^{-13}$

$[X^{2-}] = 100 \times 10^{-15}$

72. The osmotic pressure of a living cell in 12 atm at 300 K. The strength of sodium chloride solution that is isotonic with the living cell at this temperature is _____ g L^{-1}. (Nearest integer)

Given : R = 0.08 L atm K^{-1} mol^{-1}

Assume complete dissociation of NaCl

(Given : Molar mass of Na and Cl are 23 and 35.5 g mol^{-1} respectively.)

Ans. (15)

Sol. $\pi = iCRT$

$12 = 2 \times C \times 0.08 \times 300$

$12 = 2 \times C \times 24$

$C = \dfrac{1}{4}$ mole/L

then strength of NaCl solution

$= \dfrac{1}{4} \times 58.5$ g/L

$= 14.625$ g/L

$= 15$ g/L

73. A substance 'X' (1.5 g) dissolved in 150 g of a solvent 'Y' (molar mass = 300 g mol^{-1}) led to an elevation of the boiling point by 0.5 K. The relative lowering in the vapour pressure of the solvent 'Y' is _____ $\times 10^{-2}$. (Nearest integer)

[Given : K_b of the solvent = 5.0 K kg mol^{-1}]

Assume the solution to be dilute and no association or dissociation of X takes place in solution.

Ans. (3)

Sol. $\Delta T_b = i \times K_b \times m$

$0.5 = i \times m \times 5$

$i \times m = \dfrac{0.5}{5} = 0.1$

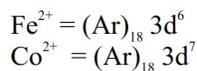$i \times a = \dfrac{15}{1000}$

(where a = moles of solute)

Now,

$\dfrac{P_o - P_S}{P^o} = i\, X_{solute} = i \times \dfrac{a}{a + \dfrac{150}{300}}$

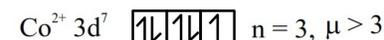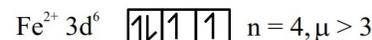$= i \times \dfrac{a}{1/2} = \dfrac{15/1000}{1/2} = \dfrac{30}{1000} = 3 \times 10^{-2} = 3$

74. Identify the metal ions among Co^{2+}, Ni^{2+}, Fe^{2+}, V^{3+} and Ti^{2+} having a spin-only magnetic moment value more than 3.0 BM. The sum of unpaired electrons present in the high spin octahedral complexes formed by those metal ions is _____.

Ans. (7)

Sol. $V^{3+} = (Ar)_{18}\, 3d^2$
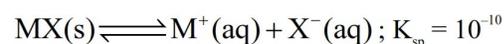$Ti^{2+} = (Ar)_{18}\, 3d^2$
$Ni^{2+} = (Ar)_{18}\, 3d^8$
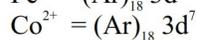
$Fe^{2+} = (Ar)_{18}\, 3d^6$
$Co^{2+} = (Ar)_{18}\, 3d^7$
Only for Fe^{2+} and Co^{2+} μ is more than 3.0 B.M.

Fe^{2+} $3d^6$ [↑↓|↑|↑][↑|↑] $n = 4, \mu > 3$

Co^{2+} $3d^7$ [↑↓|↑↓|↑][↑|↑] $n = 3, \mu > 3$

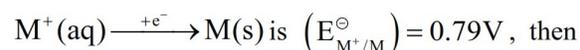$\therefore$ Number of unpaired electrons $= 4 + 3 = 7$

75. MX is a sparingly soluble salt that follows the given solubility equilibrium at 298 K

$MX(s) \rightleftharpoons M^+(aq) + X^-(aq)$; $K_{sp} = 10^{-10}$

If the standard reduction potential for

$M^+(aq) \xrightarrow{+e^-} M(s)$ is $\left(E^{\ominus}_{M^+/M}\right) = 0.79V$, then

the value of the standard reduction potential for the metal/metal insoluble salt electrode $E^{\ominus}_{X^-/MX(s)/M}$ is _____ mV. (nearest integer)

[Given: $\dfrac{2.303RT}{F} = 0.059V$]

Ans. (200)

Sol. $E^o_{X^-/MX(s)/M} = E^o_{M^+/M} + \dfrac{0.0591}{n} \log K_{sp}$

$= 0.79 + \dfrac{0.059}{1} \log 10^{-10}$

$= 0.79 - 0.59$

$= 0.20$ V $= 200$ mV